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Hu, Hao; Du, Jiaoman; Li, Xiang; Shang, Changjing; Shen, Qiang

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# Risk models for hazardous material transportation subject to weight variation considerations

Hao Hu, Jiaoman Du, Xiang Li\*, Changjing Shang, and Qiang Shen

**Abstract**—Reasonable risk models in hazardous material transportation are of practical significance, for safeguarding the lives and properties, protecting the natural environment, and facilitating the sustainable development. The existing risk models can be classified into summation risk model and maximum risk model, which result in over-reliance on overall risk or local risk. For overcoming these problems, we present two novel risk models considering different aggregation methods on local risks. The first model is supported by ordered weighted averaging (OWA) operator, which assigns the weights according to the position of the segment risk in the process of risk aggregation, and the second model is supported by state variable weight (SVW) vector, which adjusts the weights on segments according to the change of segment risk values. Generally speaking, OWA risk model is used under the situation with complete weighting information, while SVW risk model could be used under the situation with incomplete weighting information. Based on the analysis for variable weight mechanism, we show that both models could effectively balance the overall risk with the local risks assisted by weights variety. Numerical experiments are provided to illustrate the validity of the proposed risk models.

**Index Terms**—Risk model, Hazardous material transportation, Ordered weighted averaging operator, State variable weight vector.

## I. INTRODUCTION

Hazardous material (hazmat) transportation is a hot topic in the fields of public security and environmental protection. Generally speaking, the accident probability of hazmat transportation is very low, but the accident consequences may be extremely damaging. From 2013 to 2017, there were only 356 hazmat transportation accidents in China, but the accidents caused 855 deaths, 2980 injuries and huge economic losses [1]. For example, on March 19, 2016, a transport explosion accident of an oil tank truck in Beijing-Hong Kong-Macao Expressway caused 5 deaths and 21 injuries; on May 23, 2017, another transport explosion accident in Zhangjiakou-Shijiazhuang Expressway caused 15 deaths and 42 million RMB direct economy loss. Therefore, how to effectively

reduce the hazmat transportation risk has become an important and urgent subject, which has attracted great attention from both scholars and practitioners. In addition, globally, governments are also concerned with this issue, and have taken significant measures to reduce the hazmat transportation risk, such as proper driver training, enhanced vehicle maintenance, and careful emergency response planning.

In academia, risk evaluation associated with hazmat transportation has been widely investigated from different perspectives. One of the most commonly used, as proposed by Batta and Chiu [2], is expected risk model, which is defined as the product of accident probability and accident consequence. In that model, accident probabilities are obtained by calculating the historical frequencies of truck accidents, while accident consequences are the total numbers of influenced people within a certain range measured. Saccomanno and Chan [3] introduced an incident probability model, which assumes that the population densities within the affected areas are equal such that the incident probability is proportional to the risk measure. ReVelle et al. [4] suggested a population exposure model that defines the hazmat transportation risk as the sum of affected population among all segments. Comprehensive studies of this area can be found in Erkut et al. [5]. Although the above works have been widely used in hazmat transportation, they also received criticism due to the lack of local risk evaluation. For example, the expected risk model reflects the total risk value on the overall route, but ignores the high risk on certain segments near major population centres.

In reality, when dealing with high consequence events, most decision makers concern about not only overall risk, but also local risk. In addressing this issue, Abkowitz et al. [6] proposed a perceived risk model to avoid the larger population centers in the route selection process. The model replaces the accident consequences in the conventional expected risk model by an exponential function of population, aiming to effectively reduce the local risk at segments with high population along the transportation route. Erkut and Ingolfsson [7] described an alternative model, which minimizes the maximum population exposure. The model focuses on the maximum influenced population among segments along the transportation route, rather than the total influenced population on the whole route. Essentially, hazmat transportation risk along a route is the aggregation of multiple segment risks. Overall risk and local risk stand for different risk preferences of the decision makers, the former prefers to weighted averaging operators and the later max operators. In practice, both of these should be considered in the risk model. In this paper, we use two classical information aggregation methods to achieve a balance

Hao Hu is with the School of Economics and Management, Shijiazhuang Tiedao University, Shijiazhuang 050000, China.

Jiaoman Du is a PhD candidate in Department of Science and Engineering, Hosei University, Tokyo 184-8584, Japan.

Xiang Li is with the School of Economics and Management, Beijing University of Chemical Technology, Beijing 100029, China, and also with the Department of Computer Science, Aberystwyth University, Aberystwyth SY23 3DB, U.K. (Corresponding Author, E-mail: lixiang@mail.buct.edu.cn).

Changjing Shang and Qiang Shen are with the Department of Computer Science, and with the Department of Computer Science, Aberystwyth University, Aberystwyth SY23 3DB, U.K.

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between overall risk and local risk, including ordered weighted averaging (OWA) operator and state variable weight (SVW) vector.

The OWA operator is an extension of the weighted averaging operator, which was first conceived by Yager [8], and has been employed in many areas such as decision-making [9–11], data analysis [12], mathematical programming [13], and so on. In the field of risk evaluation, OWA operator also gains significant attention. For example, Hermans et al. [14] developed a road safety risk model based on OWA operator. In that model, the weights are assigned according to the ranking of local indicator values affecting the road safety risk. The higher priority the local indicator value, the greater the corresponding weight. Then the overall risk is obtained by integrating the local indicator values and OWA weighting vector. Similarly, based on OWA operator, Liu [15] presented a risk model for the high-tech project investment, and Wang and Chen [16] presented a health risk model. Although the OWA operator has been widely applied in the field of risk evaluation, there is no application on hazmat transportation risk evaluation. In order to highlight the potential impact of serious segment risks on route risk, we propose an OWA risk model for hazmat transportation, which will assign constant weights to segment risks according to the ranking of their values.

The concept of variable weights was originally presented by Wang [17] to overcome the drawbacks of the constant weighted averaging operator. Li [18, 19] illustrated the reasonability and necessity of adopting variable weights in decision-making, and gave the axiomatic definition of SVW vector. From then on, SVW vector was widely applied in decision-making [20–22]. In the field of risk evaluation, Zhang et al. [23] put forward an equipment failure risk model, in which SVW vectors are assigned to the local risk factors that may cause equipment failure. The weights associated with serious local risks are set to increase in an effort to obtain a new balance with the overall risk. In addition, Wu et al. [24] proposed another SVW risk model for coal-floor water-inrush, and Wang et al. [25] presented one for backfilling pipeline wear. Although the SVW vector has been applied in the field of risk evaluation, there is no application on hazmat transportation risk evaluation. In order to accurately characterize the impact of segment risks on route risk, we propose a novel SVW risk model for hazmat transportation, which will assign variable weights to the segment risks.

In short, in an attempt to overcome the shortcomings of the traditional risk models (over-reliance on overall risk or local risk), we propose herein two improved hazmat transportation risk models, based on OWA operator and SVW vector respectively. The main contributions of this study are: (i) we examine a representative problem case, and analyze the shortcomings of the traditional risk model in hazmat transportation from a new perspective of attribute weighting; (ii) under the situation of complete weight information, an OWA risk model for hazmat transportation is constructed, which assigns the weights according to the positions of the state values in the aggregation process, so that the severity of local risk can be reasonably reflected by the given weights; and

(iii) under the situation of incomplete weight information, a new SVW risk model is formulated, which adjusts the variable weights on segment risks according to the change of risk state values, so as to increase the proportion of serious local risk in the overall risk.

The remainder of this study is organized as follows. The next section reviews the literature regarding risk models of hazmat transportation. In Section III, mathematical preliminaries on OWA operator and SVW vector are given. Section IV presents the novel risk definitions and mathematical formulations. In Section V, we show the model application with a case study. Finally, Section VI provides a conclusion and future research directions.

## II. LITERATURE REVIEW

We present the review by surveying the existing literature that span two major threads of research: risk assessment for hazmat transportation, and risk preferences for decision maker, in an effort to explore their relationships. In hazmat transportation, we focus primarily on the applications in road transportation without covering railroad, maritime or pipeline movements (which are topics beyond the scope of the work to be presented later). Finally, we explore the feasibility of applying information integration operators to risk assessment.

In the hazmat transportation risk assessment, there are two main risk preferences: overall risk and local risk. In the early stages of risk evaluation research, multiple summation risk models were proposed to develop a potentially effective measure for minimizing the overall risk. For example, Batta and Chiu [2] considered an expected risk model, using the summation of segment risks defined by the product of accident probability and accident consequence as the risk measure. Saccomanno and Chan [3] provided an incident probability model which defined risk by the summation of segment accident probability during hazmat transportation without considering the accident consequence. This method appears to be particularly suitable for those areas less affected by hazmat and with high accident frequency. ReVelle et al. [4] presented a population exposure model for the event with relatively low probability, perceiving the population exposed to hazmat is more significant than accident probability, which defines risk as the summation of affected population among all segments. The above approaches focus on the overall risk, but ignore the disastrous consequences in the local risk, which may give rise to a biased output. For example, suppose that there are two routes and each one contains 11 segments. In the first route, segment 1 has local risk value 100, while the others have no risk. In the second route, all segments have the same local risk value 10. From the perspective of overall risk minimization, the first route should be selected, but its local risk is significantly higher than the second route.

In order to highlight the role of local risk in risk evaluation, Abkowitz et al. [6] introduced a perceived risk model which incorporated the public risk attitude into the model by adding a risk preference  $\alpha$  as an exponent of segment accident consequence. However, the interpretation of the exponent  $\alpha$  is not straightforward. Erkut and Ingolfsson [7] considered different

ways of emphasizing local risks, and presented a maximum population exposure model for catastrophe avoidance, in an effort to minimize the maximum number of exposed people among segments along the route rather than minimizing the total number of exposed people. The above methods emphasize the local risk in risk evaluation, but may generate a route with higher overall risk. Let us reconsider the example in above paragraph with local risk value 90 at all segments in the second route. From the perspective of local risk minimization, the second route should be selected, but its overall risk is significantly higher than the first route.

In order to obtain the balance between overall risk and local risk, the Mean-Variance model and Value-at-Risk model, commonly applied in portfolio selection, have been used for hazmat transportation risk evaluation. Erkut and Ingolfsson [7] simultaneously considered expected value and variance of the number of people affected by an incident, in which the expectation and variance are used as indicators to characterize the overall risk and local risk, respectively. Based on such a multi-objective model, the optimal route can be identified by adapting either Pareto optimization or weighted average operation. However, this model depends on large quantities of reliable and consistent data, which is not consistent with the low-probability characteristic of hazmat transportation accidents. Kang et al. [26] proposed a Value-at-Risk model, aiming to confine the worst risk by a factor of confidence level determined by the decision maker's preference. Tournazis and Kwon [27] applied the concept of conditional Value-at-Risk to plan the hazmat transportation route. Unlike Value-at-Risk model, it mainly concerns on the long tail of the risk distribution in order to avoid catastrophe local risk. The above methods take into account both overall risk and local risk based on the assumption that incident consequence is random variable. However, the randomness assumption may not hold since the incident frequency is generally not enough to generate the distribution function.

As applications of these risk models, more and more hazmat transportation optimization methods are proposed recently, among which the main objective is to minimize the transportation risk. On the one hand, numerous studies select overall risk as the objective function for hazmat transportation optimization models. For example, Ma et al. [28] proposed a multi-objective chance-constrained programming model for hazmat transportation, which selects the total number of population exposures as overall risk. In order to minimize the expected number of exposed people along the route, Wei et al. [29, 30] proposed the credibilistic chance-constrained programming model and credibilistic expected value model for hazmat transportation, respectively. Du et al. [31, 32] selected the expected value of affected population to describe the overall risk in hazmat vehicle routing problem, and designed different fuzzy simulation-based heuristic algorithms. Ma and Li [33] proposed a two-stage stochastic programming model for hazmat supply chain network considering reward-penalty mechanism, in which the transportation risk is defined as the total number of population exposure along the route. Hu et al. [34] proposed a credibilistic goal programming model to minimize the positive deviations of both expected risk and

expected cost. On the other hand, some studies select the local risk as a constraint for hazmat transportation optimization models. For example, Bronfman et al. [35] maximized the weighted distance between the route and its closest vulnerable centre to minimize the catastrophe consequences for exposed population; Garrido and Bronfman [36] introduced a multi-product multi-shipment hazmat routing model with equity constraints, aiming to minimize the conditional expectation of the catastrophic accident consequence. Ma et al. [37–39] developed the multi-objective robust optimization model with maximum local risk constraint for hazmat transportation. Other studies on hazmat transportation risk may be found in literature [40–43].

Employing a weighting operator to formulate the risk model is another efficient method to balance the overall risk and local risk, and has been mentioned by Erkut and Verter [44], but has not been implemented in the existing literature. The OWA operator and SVW operator are two commonly used information aggregation operators in literature. The OWA operator was introduced by Yager [8] to support aggregation lying between the max and average operators, and has been rapidly developed since its appearance [45–47]. The SVW vector was proposed by Wang [17] to overcome the drawbacks of the constant weighted averaging operator, and has been widely investigated from different perspectives [22, 48]. In this paper, we will define two new risk models based on OWA operator and SVW vector respectively, which are proved to be able to make better balance between overall risk and local risk.

### III. PRELIMINARIES

In this section, we present the basic definitions and properties for OWA operator and SVW vector. The seminal work on OWA operators was introduced by Yager [8], in which the fundamental step is reordering, i.e., rearranging the input arguments to a system model in a descending order.

*Definition 3.1:* (Yager, [8]) A  $n$ -dimensional OWA operator is a mapping  $\phi : R^n \rightarrow R$ , which has an associated weight vector  $\mathbf{W} = (w_1, w_2, \dots, w_n)$  satisfying  $w_1 + w_2 + \dots + w_n = 1$  and  $w_i \in [0, 1]$ , such that

$$\phi(\mathbf{X}) = \phi(x_1, x_2, \dots, x_n) = \sum_{i=1}^n w_i y_i,$$

where  $y_i$  is the  $i$ th largest of  $x_1, x_2, \dots, x_n$ .

*Proposition 3.1:* Let  $\phi : R^n \rightarrow R$  be a  $n$ -dimensional OWA aggregation operator with decreasing weights  $w_1 \geq w_2 \geq \dots \geq w_n$ . Then we have

$$\frac{1}{n} \cdot \sum_{i=1}^n x_i \leq \phi(x_1, x_2, \dots, x_n) \leq \max_{1 \leq i \leq n} \{x_i\}.$$

*Proof:* Compared with  $1/n$ , let  $\Delta_i$  be the increment for weight  $w_i$ , which can be expressed as

$$\Delta_i = w_i - 1/n, \quad i = 1, 2, \dots, n.$$

It is clear that

$$\Delta_1 + \Delta_2 + \dots + \Delta_n = 0.$$

Since the non-negative weights  $w_1, w_2, \dots, w_n$  are a decreasing sequence with  $w_1 + w_2 + \dots + w_n = 1$ , the weight increments

$\Delta_1, \Delta_2, \dots, \Delta_n$  are a decreasing sequence, and there exists a positive integer  $i_0$  such that  $\Delta_{i_0} \geq 0$  and  $\Delta_{i_0+1} < 0$ . Then, on the one hand, we have

$$\begin{aligned} \phi(x_1, x_2, \dots, x_n) &= \sum_{i=1}^{i_0} \left(\frac{1}{n} + \Delta_i\right) y_i + \sum_{i=i_0+1}^n \left(\frac{1}{n} + \Delta_i\right) y_i \\ &= \sum_{i=1}^n \frac{y_i}{n} + \sum_{i=1}^{i_0} \Delta_i y_i + \sum_{i=i_0+1}^n \Delta_i y_i \\ &\geq \sum_{i=1}^n \frac{y_i}{n} + \sum_{i=1}^{i_0} \Delta_i y_{i_0} + \sum_{i=i_0+1}^n \Delta_i y_{i_0} \\ &= \sum_{i=1}^n \frac{x_i}{n}. \end{aligned}$$

On the other hand, we have

$$\phi(x_1, x_2, \dots, x_n) = \sum_{i=1}^n w_i y_i \leq \sum_{i=1}^n w_i y_1 = \max_{1 \leq i \leq n} \{x_i\}.$$

The proof is complete.

The OWA operator can be regarded as a special form of variable weighted averaging operator, which was first introduced by Wang [17] to entail more flexibility in modeling system analysis, especially in decision making. Li [18] established the axiomatic definition for variable weights, and presented the axiomatic definition for SVW vector as follows.

**Definition 3.2:** (Li, [18]) A  $n$ -dimensional variable weights vector with reward (penalty) is a mapping

$$\begin{aligned} V : [0, 1]^n &\rightarrow [0, 1]^n \\ X &\mapsto V(X) = (v_1(X), v_2(X), \dots, v_n(X)) \end{aligned}$$

which satisfies axioms:

- (i)  $v_1(X) + v_2(X) + \dots + v_n(X) = 1$ ;
- (ii)  $v_i(X)$  is continuous with respect to  $x_j$  ( $i, j = 1, 2, \dots, n$ );
- (iii)  $v_i(X)$  is monotonically increasing (decreasing) with respect to  $x_i$  ( $i = 1, 2, \dots, n$ );

where  $X = (x_1, x_2, \dots, x_n)$  is a state vector.

**Definition 3.3:** (Li, [18]) A  $n$ -dimensional state variable weight vector with reward (penalty) is a mapping

$$\begin{aligned} S : [0, 1]^n &\rightarrow [0, 1]^n \\ X &\mapsto S(X) = (S_1(X), S_2(X), \dots, S_n(X)) \end{aligned}$$

which satisfies axioms:

- (i)  $x_i \leq x_j \Rightarrow S_i(X) \leq S_j(X)$ ;
- (ii)  $S_i(X)$  is continuous with respect to  $x_j$  ( $i, j = 1, 2, \dots, n$ );

(iii)  $w_i S_i(X) \times \left[ \sum_{i=1}^n w_i S_i(X) \right]^{-1}$  is continuous in  $n$ -dimensional space and monotonically increasing (decreasing) with respect to  $x_i$  ( $i = 1, 2, \dots, n$ ), where  $(w_1, w_2, \dots, w_n)$  is a constant weight vector.

Note that a  $n$ -dimensional variable weight vector can be generated by the normalized Hardarmard product of a constant

weight vector  $W$  and a SVW vector  $S(X)$ , which is written as

$$\begin{aligned} V(X) &= (v_1(X), v_2(X), \dots, v_n(X)) \\ &= \frac{(w_1 S_1(X), w_2 S_2(X), \dots, w_n S_n(X))}{\sum_{i=1}^n w_i S_i(X)}. \end{aligned} \quad (1)$$

In order to handle the hazmat transportation problem, the domain of mapping (1) is extended to  $[0, +\infty]$ . Following the principle of variable weights, a variable weighted averaging operator is a mapping  $\psi : [0, +\infty]^n \rightarrow [0, +\infty]$ , which can be written in the following form

$$\psi(X) = \sum_{i=1}^n v_i(X) x_i.$$

In this study, we only discuss the SVW vector with reward.

**Proposition 3.2:** (Li and Hao, [48]) Let  $\psi : [0, +\infty]^n \rightarrow [0, +\infty]$  be a variable weighted averaging operator based on SVW vector with reward  $(S_1(X), S_2(X), \dots, S_n(X))$ . Then we have

$$\frac{1}{n} \cdot \sum_{i=1}^n x_i \leq \psi(x_1, x_2, \dots, x_n) \leq \max_{1 \leq i \leq n} \{x_i\}.$$

**Example 3.1:** The commonly used SVW vectors with reward are listed as follows:

- (i) Power function type:  $S_i(X) = x_i^\alpha$ ;
- (ii) Exponential type:  $S_i(X) = e^{\alpha(x_i - \bar{x})}$ ;

where  $\alpha > 0$ ,  $\bar{x} = (x_1 + x_2 + \dots + x_n)/n$  and  $i = 1, 2, \dots, n$ . For example, in case of  $\alpha = 1$ , we obtain two SVW vectors with reward  $(x_1, x_2, \dots, x_n)$  and  $(e^{x_1 - \bar{x}}, e^{x_2 - \bar{x}}, \dots, e^{x_n - \bar{x}})$ .

#### IV. TRANSPORTATION RISK MODELS CONSIDERING WEIGHT VARIATIONS

Consider a hazmat transportation network  $G = (N, A)$ , where  $N$  denotes the set of nodes, and  $A$  denotes the set of edges which represent the highway segments. Let  $l = (a_0, a_1, \dots, a_n)$  be a route between an origin-destination (O-D) pair consisting of some unidirectional edges denoted by  $a_{i-1}a_i$ , where  $a_i \in N$  for all  $i = 1, 2, \dots, n$ . Denote  $R_{a_{i-1}a_i}$  as the risk at edge  $a_{i-1}a_i$  for hazmat transportation, and  $p_{a_{i-1}a_i}$  as the probability of hazmat transportation accident at edge  $a_{i-1}a_i$ . The consequence is measured by the number of exposed people with a given impact radius, and is denoted by  $c_{a_{i-1}a_i}$ . A critical issue for hazmat transportation research is how to select an optimal route to minimize the transportation risk. Note that different transportation risk models may lead to diversification of the optimal route. Generally speaking, existing definitions regarding hazmat transportation risk along route  $l$  can be classified into two types including:

- (i) Summation risk model

$$R_l = \sum_{i=1}^n R_{a_{i-1}a_i} = \sum_{i=1}^n p_{a_{i-1}a_i} c_{a_{i-1}a_i}; \quad (2)$$

- (ii) Maximum risk model

$$R_l = \max_{1 \leq i \leq n} R_{a_{i-1}a_i} = \max_{1 \leq i \leq n} \{p_{a_{i-1}a_i} c_{a_{i-1}a_i}\}, \quad (3)$$

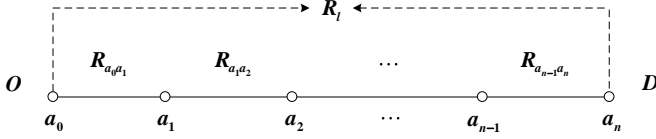


Fig. 1. Transportation risk along a route.

where  $R_l$  represents the total risk along route  $l$  (see Fig. 1).

Eq. (2) is the most popular risk measure and is also known as the expected risk model (Batta and Chiu, [2]). It states that the overall risk along route  $l$  is equal to the summation of local risks at all edges belonging to the route. The transportation model with summation risk measure is to reduce the overall risk along the route. However, a route with the minimum sum of edge risks may pass through one or more edges involving dense population or high accident probability resulting in the surge of high local risk. Therefore, this summation risk model may underestimate the importance of the maximum edge risk along the route. To address this issue, the maximum risk model is proposed to measure the transportation risk along route  $l$ . Erkut and Ingolfsson [7] defined the edge risk as the number of population exposures, and proposed a maximum population exposure model to avoid serious local risk. Nonetheless, although this model reduces the maximum local risk along the route, it may over-estimate the overall risk. Without loss of generality, for facilitating the comparison with Eq. (2), we can also define the edge risk as  $p_{a_{i-1}a_i}c_{a_{i-1}a_i}$ , which generates Eq. (3).

*Example 4.1:* Suppose that  $l_1$ ,  $l_2$  and  $l_3$  are three alternative routes for hazmat transportation, all of which contain four edges (see Fig. 2). The probability and consequence of a hazmat transportation accident at edges along routes  $l_1$ ,  $l_2$  and  $l_3$  are presented in Table I. If the summation risk model is used, we have

$$R_{l_1} = 23, R_{l_2} = 40, R_{l_3} = 24,$$

which means that the route  $l_1$  is the best choice. However, the local risk at the third edge along route  $l_1$  is extremely high, which may lead to a serious accident consequence in reality. If the maximum risk model is used instead, we have

$$R_{l_1} = 20, R_{l_2} = 10, R_{l_3} = 11,$$

which means that the route  $l_2$  is the best choice. However, the overall risk along route  $l_2$  is significantly higher than that along routes  $l_1$  and  $l_3$ . In this sense, route  $l_2$  is not a better choice actually.

In short, the summation risk model emphasizes on the overall risk along the route, while the maximum risk model pays more attention on local risk. Neither of them is able to provide a good balance between overall risk and local risks. Therefore, we need to construct a novel transportation risk model to overcome the drawbacks of these two traditional risk models.

The summation risk model can be rewritten by the arith-

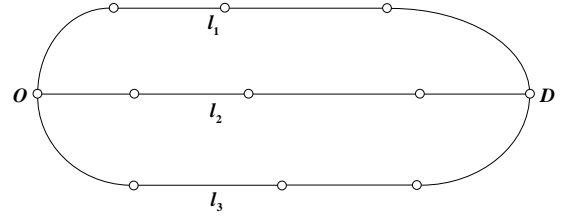


Fig. 2. Transportation routes for given O-D pair.

metic average of all edge risks as

$$R_l = \sum_{i=1}^n R_{a_{i-1}a_i} = n \cdot \sum_{i=1}^n w_i R_{a_{i-1}a_i},$$

where  $w_i = 1/n$  for all  $i = 1, 2, \dots, n$ . Thus, the model can be interpreted from the perspective of weighted averaging operator, which treats all edge risks being equally important. However, it ignores the fact that some routes may have a higher edge risk than others. In order to overcome this drawback, we extend the uniform weights to non-uniform weights with a general representation, attempting to assign higher weights to serious edge risks (see Fig. 3).

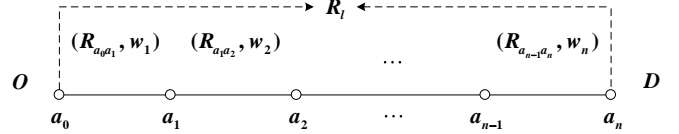


Fig. 3. Transportation risk with weights along a route.

The most important issue is to determine the optimal allocation of the weights. In order to avoid catastrophic accidents, it is necessary to attach more attention to the edges with higher local risks. For this purpose, a novel risk model based on OWA operator is herein introduced.

#### A. OWA Risk Model

In this subsection, we present an OWA risk model, and describe the common used ways for constructing the required weight vector.

*Definition 4.1:* Suppose that  $l = (a_0, a_1, \dots, a_n)$  is a route and  $w_1, w_2, \dots, w_n$  are a non-negative decreasing sequence satisfying  $w_1 + w_2 + \dots + w_n = 1$ . The OWA risk model is defined as

$$R_l = n \cdot \sum_{i=1}^n w_i \hat{R}_{a_{i-1}a_i}, \quad (4)$$

where  $\hat{R}_{a_{i-1}a_i}$  is the  $i$ th largest value in the set  $\{R_{a_0 a_1}, R_{a_1 a_2}, \dots, R_{a_{n-1} a_n}\}$  for all  $i = 1, 2, \dots, n$ .

Eq. (4) assigns the weight  $w_i$  to the edge risk of the  $i$ th position from top to down in a risk aggregation process. In order to strengthen the impact of higher edge risk on the whole route risk, the weight sequence  $w_1, w_2, \dots, w_n$  is set in descending order, which is essentially different from the

TABLE I Total transportation risk at edges defining  $l_1, l_2, l_3$ 

Transportation route	Accident probability ( $\times 10^{-5}$ )	Accident consequence ( $\times 10^5$ )	Edges risks
$l_1$	(2, 1, 1, 4)	(0.5, 1, 20, 0.25)	(1,1,20,1)
$l_2$	(2, 4, 8, 2)	(5, 2.5, 1.25, 5)	(10,10,10,10)
$l_3$	(4, 2, 6, 2)	(1.25, 2.5, 0.5, 5.5)	(5,5,3,11)

arithmetic averaging method. In this case, the OWA risk model obtains a compromise between overall risk and local risks.

*Proposition 4.1:* Let  $W = (w_1, w_2, \dots, w_n)$  be a weight vector. Then we have:

- (i) the OWA risk model degenerates to the summation risk model when  $W = (1/n, 1/n, \dots, 1/n)$ ;
- (ii) the OWA risk model is equivalent to the maximum risk model when  $W = (1, 0, \dots, 0)$ .

*Proof:* The proof is trivial and omitted.

Different risk models can be generated by changing the weight vector. In theory, there are infinite ways to construct the weight vector associated with OWA risk model, among which the arithmetical progression and geometric progression are two particular ones, as illustrated in the following examples.

*Example 4.2:* Suppose that the weights  $w_1, w_2, \dots, w_n$  in Eq. (4) obey an arithmetical progression and  $d$  is the common difference. Then, the weight vector can be expressed as  $(w_1, w_1 - d, \dots, w_1 - (n-1)d)$ . Since the weights are normalized (i.e.,  $w_1 + w_2 + \dots + w_n = 1, w_i \in [0, 1]$ ), we have

$$w_1 + w_2 + \dots + w_n = nw_1 - n(n-1)d/2 = 1,$$

which implies  $w_1 = [(n^2 - n)d + 2]/2n$ . When  $n = 1$ , we have  $w_1 = 1$ . When  $n > 1$ , we have

$$w_i = \frac{(n^2 + n - 2ni)d + 2}{2n}, \quad i = 2, 3, \dots, n.$$

Since the weights and common difference are both non-negative, we have

$$w_n = \frac{(n - n^2)d + 2}{2n} \geq 0 \quad \text{and} \quad d \geq 0,$$

which leads to  $0 \leq d \leq 2/[n(n-1)]$ .

*Example 4.3:* Suppose that the weights  $w_1, w_2, \dots, w_n$  in Eq. (4) obey a geometric progression and  $q$  is the common ratio. Then, the weight vector can be expressed as  $(w_1, w_1q, \dots, w_1q^{n-1})$ . Since the weights are normalized, the following results can be obtained: (i) if  $q = 1$ , we have

$$w_i = 1/n, \quad i = 1, 2, \dots, n;$$

(ii) if  $q \neq 1$ , we have

$$w_1 + w_2 + \dots + w_n = w_1 \cdot \frac{1 - q^n}{1 - q} = 1,$$

which implies

$$w_i = \frac{q^{i-1} - q^i}{1 - q^n}, \quad i = 1, 2, \dots, n.$$

Since the weights and common ratio are both non-negative, we have

$$w_n = \frac{q^{n-1} - q^n}{1 - q^n} > 0 \quad \text{and} \quad q > 0,$$

which implies  $0 < q < 1$ .

*Example 4.4:* Let us reconsider Example 4.1 by taking the OWA risk model with weight vector  $(9/24, 7/24, 5/24, 3/24)$ . If the OWA risk model is used, we have

$$R_{l_1} = 32.5, R_{l_2} = 40, R_{l_3} = 28,$$

which means that the route  $l_3$  is the best choice. Compared with route  $l_2$ , the overall risk along route  $l_3$  is decreased by 40%, and compared with route  $l_1$ , the maximum edge risk along route  $l_3$  is decreased by 45%.

In general, compared with the summation risk model and maximum risk model, the OWA risk model achieves a better balance between overall risk and local risks by highlighting the impact of serious edge risks. In practice, decision makers could select different weight vectors freely with respect to the category of hazmats. However, such an OWA risk model is commonly applied under the situation with complete weighting information. When the weights information is incomplete, the OWA risk model does not work, and we propose another risk model based on SVW vector as described below.

#### B. SVW Risk Model

In this subsection, we present a SVW risk model, and describe the common used methods for constructing the required SVW vectors.

*Definition 4.2:* Suppose that  $l = (a_0, a_1, \dots, a_n)$  is a route and  $\mathbf{R} = (R_{a_0a_1}, R_{a_1a_2}, \dots, R_{a_{n-1}a_n})$  is a risk state vector along route  $l$ . Let  $(w_1, w_2, \dots, w_n)$  be a non-negative weight vector satisfying  $w_1 + w_2 + \dots + w_n = 1$ . The SVW risk model is defined as

$$R_l = n \cdot \sum_{i=1}^n v_i(\mathbf{R}) R_{a_{i-1}a_i}, \quad (5)$$

where  $v_i(\mathbf{R}) = w_i S_i(\mathbf{R}) \times \left[ \sum_{j=1}^n w_j S_j(\mathbf{R}) \right]^{-1}$ , and  $(S_1(\mathbf{R}), S_2(\mathbf{R}), \dots, S_n(\mathbf{R}))$  is a SVW vector with reward.

Compared with the summation risk model, Eq. (5) converts constant weights into variable weights with reward. Following Definition 3.3, under the action of SVW vector with reward, there is a positive correlation between variable weight  $v_i(\mathbf{R})$  and edge risk  $R_{a_{i-1}a_i}$ , which means that the greater the edge risk, the greater the increment of variable weight value. Therefore, the SVW risk model obtains another balance between overall risk and local risks along a route. Since different SVW vectors have different capabilities of adjusting weight values, we can obtain different risk results by changing the SVW vector. Indeed, it can be readily established that a relationship among SVW risk model, OWA risk model and two traditional risk models using an appropriate SVW vector as follows.

*Proposition 4.2:* Let  $\mathbf{W} = (w_1, w_2, \dots, w_n)$  be a weight vector. Then we have:

- (i) the SVW risk model degenerates to the summation risk model when  $\mathbf{W} = (1/n, 1/n, \dots, 1/n)$ , and  $S_i(\mathbf{R}) = S_j(\mathbf{R})$  ( $\forall i, j = 1, 2, \dots, n$ );
- (ii) the SVW risk model is equivalent to the maximum risk model when  $w_{j^*} = 1$ , where  $j^* = \arg \max_{1 \leq i \leq n} (R_{a_{i-1}a_i})$ .

*Proof:* The proof is trivial and omitted.

**Proposition 4.3:** Let  $(w_1, w_2, \dots, w_n)$  be the weight vector in OWA risk model, and  $(w_1^*, w_2^*, \dots, w_n^*)$  be the constant weight vector in SVW risk model. Then, the SVW risk model degenerates to the OWA risk model when  $S_{k_j}(\mathbf{R}) = w_j/w_{k_j}^*$  with  $0 < w_{k_j}^* \leq (w_j \cdot w_{k_{j+1}}^*)/w_{j+1}$  and

$$k_j = \begin{cases} \arg \max_{1 \leq i \leq n} (R_{a_{i-1}a_i}), & \text{if } j = 1, \\ \arg \max_{1 \leq i \leq n, i \notin \bigcup_{t=1}^{j-1} \{k_t\}} (R_{a_{i-1}a_i}), & \text{if } j = 2, 3, \dots, n. \end{cases}$$

*Proof:* It is straightforward to establish that the vector  $(w_1/w_{k_1}^*, w_2/w_{k_2}^*, \dots, w_n/w_{k_n}^*)$  obeys Definition 3.3, which means it can be regarded as a SVW vector with reward. Then we have

$$\begin{aligned} R_l &= n \cdot \sum_{i=1}^n \frac{w_i^* S_i(\mathbf{R})}{\sum_{j=1}^n w_j^* S_j(\mathbf{R})} R_{a_{i-1}a_i} = n \cdot \sum_{i=1}^n \frac{w_{k_i}^* S_{k_i}(\mathbf{R})}{\sum_{j=1}^n w_{k_j}^* S_{k_j}(\mathbf{R})} R_{a_{k_{i-1}}a_{k_i}} \\ &= n \cdot \sum_{i=1}^n \frac{w_{k_i}^* \frac{w_i}{w_{k_i}^*}}{\sum_{j=1}^n \frac{w_j}{w_{k_j}^*}} R_{a_{k_{i-1}}a_{k_i}} = n \cdot \sum_{i=1}^n w_i R_{a_{k_{i-1}}a_{k_i}} \\ &= n \cdot \sum_{i=1}^n w_i \hat{R}_{a_{i-1}a_i} \end{aligned}$$

where  $\hat{R}_{a_{i-1}a_i}$  is the  $i$ th largest value in the set  $\{R_{a_0a_1}, R_{a_1a_2}, \dots, R_{a_{n-1}a_n}\}$ . The proof is complete.

As the weight information is generally incomplete, we could assign equal weight  $1/n$  to each edge risk. Under the action of SVW vector with reward, Eq. (5) increase the weight with higher edge risk. Therefore, the SVW risk model could be used under the situation of incomplete weight information. The key issue of such a model is to determine the optimal form of the SVW vector for use. The following examples show two types of SVW vectors with reward, which are of practical significance.

**Example 4.5:** Let  $\mathbf{R} = (R_{a_0a_1}, R_{a_1a_2}, \dots, R_{a_{n-1}a_n})$  be a risk state vector along the route. The SVW vector with reward in Eq. (5) can be constructed as follows:

- (i) Power exponential function type

$$S_i(\mathbf{R}) = R_{a_{i-1}a_i}^\alpha, \quad \alpha > 0, i = 1, 2, \dots, n,$$

which indicates a power exponential relationship between the  $i$ th edge risk and its SVW;

- (ii) Exponential type

$$S_i(\mathbf{R}) = e^{\alpha(R_{a_{i-1}a_i} - \bar{R})}, \quad \alpha > 0, i = 1, 2, \dots, n,$$

where  $\bar{R} = (R_{a_0a_1} + R_{a_1a_2} + \dots + R_{a_{n-1}a_n})/n$ , which indicates an exponential relationship between the  $i$ th edge risk and its SVW.

**Example 4.6:** Let us reconsider Example 4.1 by taking the SVW risk model. If the weights information is incomplete, set the constant weight vector as  $(1/4, 1/4, 1/4, 1/4)$ , and define the SVW vector with reward as

$$S_i(\mathbf{R}) = R_{a_{i-1}a_i}^\alpha, \quad i = 1, 2, 3, 4,$$

where  $\alpha = 0.5$ . Then we have

$$R_{l_1} = 49.5, R_{l_2} = 40, R_{l_3} = 26.9,$$

which means that the route  $l_3$  is the best choice. Therefore, compared with the summation risk model and maximum risk model, the SVW risk model achieves a better balance between overall risk and local risks.

In reality, there are infinite ways to construct SVW vectors. For example, we can combine the given SVW vectors, including those of the aforementioned types, to produce new SVW vectors. In reality, the selection of a weight vector should be made on the basis of informed hazmat classification.

## V. APPLICATION TO HAZMAT RISK MODELING AND CASE STUDY

In general, the route optimization problems concerning on hazmat transportation involve: shortest path problem (SPP), travel salesman problem (TSP) and vehicle routing problem (VRP). As our study mainly focuses on the risk evaluation for hazmat transportation rather than route optimization, we take TSP as an example to illustrate the efficiency of OWA risk model and SVW risk model. Considering a node set  $N$ , where hazmat depot is the origin  $a_0$  which also serves as the destination  $a_n$ . There are  $n-1$  hazmat retailers along the transportation route, which are denoted by  $a_i$  for  $i = 1, 2, \dots, n-1$  (See Fig. 4). The hazmat TSP can be described as the process where the hazmat vehicle departs from the depot, visits each retailer exactly once, and finally returns to the depot. The edge risk is denoted by  $R_{a_{i-1}a_i}$  where  $a_i \in N$  for all  $i = 1, 2, \dots, n$ . The objective is to determine the transportation route with minimum risk. Suppose that  $x_{ij}$  is the decision variable, such that if the edge from  $i$  to  $j$  is active, it takes value 1; otherwise, it takes value 0.

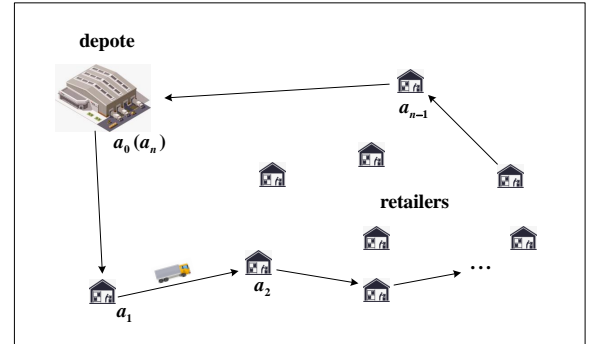


Fig. 4. Experimental transportation network.

### A. Hazmat TSP Models

Du et al. [32] proposed a TSP model for hazmat transportation problem, aiming to minimize the total transportation



risk. In this subsection, we replace their summation risk model by OWA risk model and SVW risk model respectively, and obtain two new hazmat TSP models. First, based on Eq. (4), we obtain an OWA risk-based hazmat transportation model

$$\left\{ \begin{array}{l} \min \quad R = n \cdot \sum_{i=1}^n w_i \hat{R}_{a_{i-1}a_i} \\ \text{s.t.} \quad \sum_{i \in N, i \neq j} x_{ij} = \sum_{j \in N, i \neq j} x_{ij} = 1, \quad i, j \in N \\ \sum_{i, j \in N} x_{ij} \leq |M| - 1, \quad 2 \leq |M| \leq n - 1, \quad M \subset N \\ a_i = \arg \max_{j \in N \setminus \bigcup_{k=0}^{i-1} \{a_k\}} (x_{a_{i-1}j}), \quad 1 \leq i \leq n - 1 \\ x_{ij} \in \{0, 1\}, \quad i, j \in N, \end{array} \right. \quad (6)$$

where  $\hat{R}_{a_{i-1}a_i}$  is the  $i$ th largest value in the set  $\{R_{a_0a_1}, R_{a_1a_2}, \dots, R_{a_{n-1}a_n}\}$ . The first constraint represents flow conservation, the second constraint is sub-tour elimination constraint, the third constraint represents the retailer whose service order is  $i$  along the transportation route, and the last constraint defines the domain of the decision variables.

Second, based on Eq. (5), we obtain a SVW risk-based hazmat transportation model

$$\left\{ \begin{array}{l} \min \quad R = n \cdot \sum_{i=1}^n v_i(\mathbf{R}) R_{a_{i-1}a_i} \\ \text{s.t.} \quad \sum_{i \in N, i \neq j} x_{ij} = \sum_{j \in N, i \neq j} x_{ij} = 1, \quad i, j \in N \\ \sum_{i, j \in N} x_{ij} \leq |M| - 1, \quad 2 \leq |M| \leq n - 1, \quad M \subset N \\ a_i = \arg \max_{j \in N \setminus \bigcup_{k=0}^{i-1} \{a_k\}} (x_{a_{i-1}j}), \quad 1 \leq i \leq n - 1 \\ x_{ij} \in \{0, 1\}, \quad i, j \in N, \end{array} \right. \quad (7)$$

where  $v_i(\mathbf{R}) = w_i S_i(\mathbf{R}) \times \left[ \sum_{j=1}^n w_j S_j(\mathbf{R}) \right]^{-1}$ , and  $(S_1(\mathbf{R}), S_2(\mathbf{R}), \dots, S_n(\mathbf{R}))$  is a SVW vector with reward.

### B. Numerical Experiments

In this subsection, we demonstrate the effectiveness of OWA risk and SVW risk in comparison to the traditional summation risk and maximum risk, using the TSP models (6) and (7). After that, the sensitivity analyses on common difference  $d$  and exponent  $\alpha$  are conducted.

Although TSP is a NP-hard problem (Lin and Kernighan, [49]), effective solution algorithms have been proposed and applied in practice. For a small-scale networks, we can use exhaustive method or branch and bound algorithm (Padberg and Rinaldi, [50]). For large-scale networks, heuristic techniques such as genetic algorithm (Nguyen, [51]; Hu et al., [52]), ant colony algorithm (Musa et al. [53]) and hybrid particle swarm optimization (Du et al. [32]) can be used to search satisfactory solutions. However, the search algorithm is not the focus of this study and hence is not addressed further.

As a case study to experimentally verify the proposed approaches, we consider a hazmat transportation network consisting of 1 depot and 10 retailers (See Fig. 5). The local risk at each edge are presented in Table II. For this small-scale TSP networks, we use the branch and bound algorithm to calculate the exact solution.

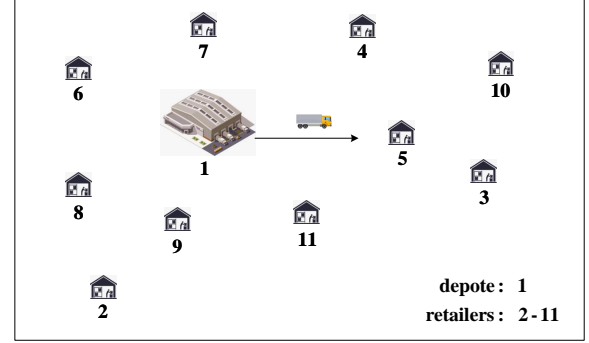


Fig. 5. Experimental transportation network.

1) *Running Traditional Risk Models*: Firstly, both summation risk model and maximum risk model are used to obtain the optimal route for this transportation network. Table III shows the results, including transportation routes and risks, where  $R_1^*$ ,  $R_2^*$ ,  $R_3^*$  and  $R_4^*$  are specified as the route risk computed using summation risk model, maximum risk model, OWA risk model, and SVW risk model, respectively. To be specific, utilizing the summation risk model, we obtain a route with the minimum overall risk of 294, which is denoted by  $l_1$ ; utilizing the maximum risk model, we obtain 68 routes with the minimum local risk of 66. For the purpose of comparison, we select two routes with the minimum and maximum overall risks from these 68 routes, which are denoted by  $l_{21}$  and  $l_{22}$ , respectively.

Comparing the routes generated by summation risk model and maximum risk model, we note that the maximum local risk along route  $l_1$  is significantly higher than that along routes  $l_{21}$  and  $l_{22}$ , reaching  $(80 - 66)/66 \times 100\% = 21.2\%$ . On the contrary, the overall risks along routes  $l_{21}$  and  $l_{22}$  are both higher than that along route  $l_1$ , reaching  $(322 - 294)/294 \times 100\% = 9.5\%$  and  $(488 - 294)/294 \times 100\% = 66\%$ , respectively. The results illustrate the defects of traditional risk models, which are not able to provide a good balance between overall risk and local risks.

2) *Running OWA Risk Model*: We run the OWA risk-based hazmat transportation model, where the weight vectors are defined by an arithmetical progression as

$$w_i = \frac{(n^2 + n - 2ni)d + 2}{2n}, \quad i = 1, 2, \dots, n,$$

with  $d = 1/[n(n - 1)]$ . The results are presented in Table III, where  $l_3$  is the route with the minimum OWA risk of 289.7.

Compared with route  $l_1$ , the overall risk along route  $l_3$  has a mere increase of  $(301 - 294)/294 \times 100\% = 2.4\%$ , while the maximum local risk is significantly decreased by  $(80 - 67)/80 \times 100\% = 16.3\%$ . Compared with routes  $l_{21}$  and  $l_{22}$ , the maximum local risk along route  $l_3$  has a slight increase of  $(67 - 66)/66 \times 100\% = 1.5\%$ , while the overall risks are reduced

TABLE II Transportation risk at each edge

	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	95	—	—	—	—	—	—	—	—	—	—
3	67	75	—	—	—	—	—	—	—	—	—
4	80	31	95	—	—	—	—	—	—	—	—
5	70	75	27	67	—	—	—	—	—	—	—
6	85	25	24	69	89	—	—	—	—	—	—
7	81	24	92	34	19	25	—	—	—	—	—
8	91	28	75	75	88	56	7	—	—	—	—
9	66	46	1	33	16	79	31	52	—	—	—
10	68	74	45	8	22	91	15	82	53	—	—
11	60	96	80	77	81	86	23	39	25	80	—

TABLE III Comparisons among four risk models

	Transportation route	$R_1^*$	$R_2^*$	$R_3^*$	$R_4^*$
$l_1$	1 → 11 → 7 → 8 → 2 → 6 → 3 → 9 → 5 → 10 → 4 → 1	294	80	290.3	338.2
$l_{21}$	1 → 11 → 8 → 7 → 5 → 10 → 4 → 2 → 6 → 3 → 9 → 1	322	66	331	361.7
$l_{22}$	1 → 11 → 8 → 6 → 2 → 4 → 7 → 5 → 3 → 10 → 9 → 1	488	66	478.9	495.6
$l_3$	1 → 11 → 8 → 7 → 6 → 2 → 4 → 10 → 5 → 9 → 3 → 1	301	67	289.7	341
$l_4$	1 → 11 → 9 → 5 → 10 → 4 → 2 → 8 → 7 → 6 → 3 → 1	313	67	317.8	338

by  $(322 - 301)/322 \times 100\% = 6.5\%$  and  $(488 - 301)/488 \times 100\% = 38.3\%$ , respectively. The results illustrate that OWA risk model is able to achieve a better balance between overall risk and local risks.

3) *Running SVW Risk Model*: We run the SVW risk-based hazmat transportation model with weight  $w_i = 1/n$  and

$$S_i(\mathbf{R}) = R_{a_{i-1}a_i}^\alpha, \quad i = 1, 2, \dots, n,$$

where  $\alpha = 0.2$ . The results are presented in Table III, where  $l_4$  is the route with the minimum SVW risk of 338.

Compared with route  $l_1$ , the overall risk along route  $l_4$  has a mere increase of  $(313 - 294)/294 \times 100\% = 6.5\%$ , while the maximum local risk is significantly decreased by  $(80 - 67)/80 \times 100\% = 16.3\%$ . Compared with routes  $l_{21}$  and  $l_{22}$ , the maximum local risk along route  $l_4$  has a slight increase of  $(67 - 66)/66 \times 100\% = 1.5\%$ , while the overall risks are reduced by  $(322 - 313)/322 \times 100\% = 2.8\%$  and  $(488 - 313)/488 \times 100\% = 35.9\%$ , respectively. The results illustrate the efficiency of the SVW risk model on balancing overall risk and local risks.

4) *Sensitivity Analysis*: Since the optimal routes generated by models (6) and (7) depend on their parameter setting in the corresponding risk models, we conduct sensitivity analyses on parameters  $d$  and  $\alpha$ . First, we show the relationship between OWA risk  $R_3^*$  and common difference  $d$  by Fig. 6, where  $0 \leq d \leq 2/[11 \times (11 - 1)] = 0.0182$ . With the uniform increase of common difference  $d$ , the OWA risk  $R_3^*$  shows a steady decreasing trend with a total decrease of 5.2% within the domain of  $d$ , which implies that model (6) is insensitive to the variation of common difference  $d$ .

Second, we summarize the effect of exponent  $\alpha$  on SVW risk model by Fig. 7. It is shown that SVW risk  $R_4^*$  has a significant increase when  $0.001 \leq \alpha \leq 20$ , but tends to be stable when  $\alpha > 20$ . Therefore, when SVW risk model is adopted for practical use, empirical selection of an appropriate exponent  $\alpha$  is necessary in order to build the risk model. Factors to be considered should include the characteristics of hazmat, preferences of decision makers and natural environment of

transportation networks. For example, if the decision-makers prefer to control the maximum local risk, they could select an  $\alpha$  in the interval  $[20, +\infty)$ ; otherwise, they could select an  $\alpha$  in the interval  $[0, 20)$ .

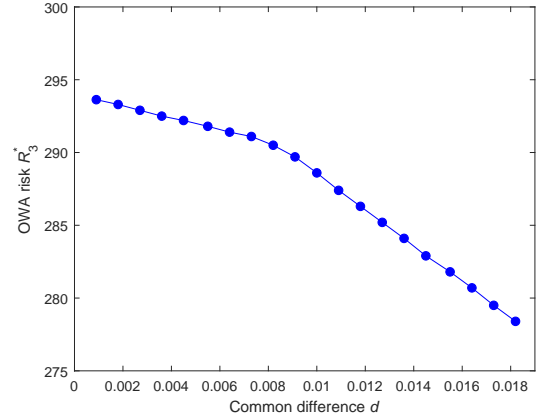


Fig. 6. Relationship between the minimum OWA risk  $R_3^*$  and common difference  $d$ .

## VI. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This study has proposed two novel risk models for hazmat transportation by taking into consideration of weight variation, aiming to make a better balance between overall risk and local risk. The OWA risk model focuses on assigning weights with respect to the ranking of the segment risks, which can be applied when weight information is complete. The SVW risk model is produced as the result of adjusting the weights at segments of a certain transportation route, with respect to the change of local risk values, which can be used under the situation of incomplete weight information. Numerical experiments have demonstrated the effectiveness of both OWA risk model and SVW risk model. In particular, the results have shown: (i) compared with the traditional summation risk model

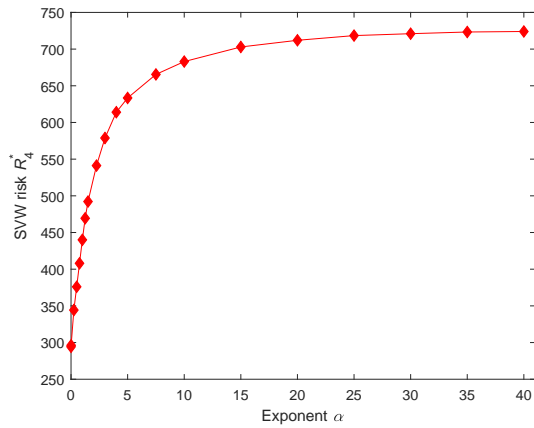


Fig. 7. Relationship between the minimum SVW risk  $R_4^*$  and exponent  $\alpha$ .

and maximum risk model, the OWA risk model is able to lower the local risk by 16.3% and the overall risk by 6.5%~38.3%; (ii) compared with the summation risk model and maximum risk model, the SVW risk model is able to lower the local risk by 16.3% and the overall risk by 2.8%~35.9%; and (iii) the sensitivity analyses on the model parameters (common difference  $d$  and exponent  $\alpha$ ) have shown the robustness of both proposed risk models.

To meet the practical transportation needs for different types of hazmats and risk preferences of decision makers, we could change the parameters in the proposed risk models to seek the corresponding optimal transportation routes. In our future studies, we plan to focus on how to select the model parameters for addressing different transportation scenarios. Second, although this work has presented a systematic case study, both novel models would benefit from being applied to more broader hazmat transportation problems, such as multiple travel salesman problems, shortest path problems, vehicle routing problems, and so on. Third, it will be valuable to take more realistic considerations concerning randomness or fuzziness in transportation environment, and deal with the uncertain hazmat transportation problem using some flexible optimization approaches, such as chance-constrained programming model (Du et al. [31]), mean-variance model (Guo et al. [54]) and mean-entropy model (Zhou et al. [55]).

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**Changjing Shang** received the Ph.D. degree in computing and electrical engineering from Heriot-Watt University, Edinburgh, U.K., in 1995.

She is currently a University Research Fellow with the Department of Computer Science, Institute of Mathematics, Physics and Computer Science, Aberystwyth University, Aberystwyth, U.K. Prior to joining Aberystwyth, she was with Heriot-Watt, Loughborough and Glasgow Universities. She has published extensively, and supervised 15 Ph.Ds/PDRAs in the areas of pattern recognition,

data mining and analysis, space robotics, and image modeling and classification.



**Hao Hu** received M.S. degree in Mathematics from Nanjing University of Science and Technology, Nanjing, China, in 2008, and the Ph.D. degree in Control Science and Engineering from Beijing University of Chemical Technology, Beijing, China, in 2018.

He is a lecturer in School of Economics and Management, Shijiazhuang Tiedao University, Shijiazhuang. His current research interests include credibility theory, fuzzy optimization, and its applications in hazardous material transport.



**Jiaoman Du** received the M.S. degree in Management Science and Engineering from Beijing University of Chemical Technology, Beijing, China, in 2016. She is currently working toward the Ph.D. degree in Department of Science and Engineering from Hosei University, Tokyo.

Her current research interests include transportation systems and optimization theory, credibility theory and so on.



**Qiang Shen** received the Ph.D. degree in computing and electrical engineering from Heriot-Watt University, Edinburgh, U.K., in 1990, and the D.Sc. degree in computational intelligence from Aberystwyth University, Aberystwyth, U.K., in 2013.

He holds the Established Chair of Computer Science and is currently Pro Vice-Chancellor for Faculty of Business and Physical Sciences, Aberystwyth University. He has authored two research monographs and approximately 400 peer-reviewed papers.

Dr. Shen was the recipient of an Outstanding Transactions Paper Award from the IEEE.



**Xiang Li** received the Ph.D. degree from Tsinghua University, Beijing, China, in 2008.

He is currently a professor with School of Economics and Management Science, Beijing University of Chemical Technology. His research interests include intelligent transportation system, optimization under uncertainty, big data analysis and so on. He has authored one book and more than 80 articles on international journals including *Applied Mathematical Modelling*, *Omega*, *Transportation Research Part B*, *Transportation Research Part C*, *Information*

*Sciences*, *IEEE Transactions on Fuzzy Systems*, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *IEEE Transactions on Intelligent Transportation Systems*, *European Journal of Operational Research*, *Computer and Industrial Engineering* and so on, which has been cited more than 1700 times on Web of Science and 3000 times on Google Scholar.

He served as the associate editor or editorial board member of *Information Sciences*, *Transportmetrica B*, *International Journal of General Systems*, *Journal of Ambient Intelligence and Humanized Computing*, *Sustainability* and so on.